Quantum-Geometrical Space

Let me say at the outset that I am not happy with this state of affairs in physical theory. The mathematical continuum has always seemed to me to contain many features which are really very foreign to physics. […] If one is to accept the physical reality of the continuum, then one must accept that there are as many points in a volume of diameter 10^{13} cm or 10^{33} cm or 10^{1000} cm as there are in the entire universe. Indeed, one must accept the existence of more points than there are rational numbers between any two points in space no matter how close together they may be. (And we have seen that quantum theory cannot really eliminate this problem, since it brings in its own complex continuum.)

Roger Penrose, On the Nature of Quantum-Geometry

The Nature of Space

I consider it quite possible that physics cannot be based on the field concept, i. e., on continuous structures. In that case nothing *remains of my entire castle in the air, gravitation theory included, [and of] the rest of modern physics.* - Einstein in a 1954 letter to Besso.

What Einstein might have been referring to is that special relativity and general relativity require that space be continuous. The axiom of continuity of space is implied by special relativity as well as most current physics theory.

Einstein understood that if the implied continuity axiom turned out not to correspond to the fundamental nature of space, his theory and all theories which are based on it would also fall apart. We disagree. Einstein's theories would still hold very well if space were discrete rather than continuous, and so would be the principle of relativity.

When considering that predictions of the relativity theories have been confirmed by countless experiments and observations, it is logical to assume that their underlying axioms must be correct, including that of space continuum which is an implicit axiom. And space continuum and space discreteness being mutually exclusive, if space were discrete, then it would follow that space continuum and theories founded on it would be wrong, right? But what if the space continuum was not fundamental? What if space only appears and behaves to be continuous at larger than the fundamental scale allowing physical theories such as the relativity theories to correctly describe systems at those scales. Then space continuum would not be an axiom in the sense we have described here, but a theorem. That could explain why general relativity can correctly describe dynamic systems at large scales while failing for systems at the fundamental scale where space would be discrete. If this were the case, then understanding how the space continuum emerges from discrete space would open the door to fundamental theories that can describe dynamics systems in discrete space while still being compatible with theories such as general relativity.

Dominant theories successfully explained and predict phenomena at scales which they observed and from which observations their theorems were derived. Space continuum is what is observed at non-fundamental scales.

Quantum-geometry dynamics postulates that space is fundamentally discrete. Specifically, that space is quantum-geometrical, that is: Quantum-geometrical space is formed by fundamental particles we call $preons^{(-)}$ (symbol $\boldsymbol{p}^{(-)}$) and is dimentionalized by the repulsive force acting between them. Thus according to QGD, spatial dimensions are emergent properties of $preons^{(-)}$, hence dimentionalized space is not fundamental.

The interaction between any two $preons^{(-)}$ is the fundamental unit of the force acting between them which because it is repulsive, we will call n-gravity (symbol $\boldsymbol{g}^{-}$).

It is important here to remind the reader that what exists between two $preons^{(-)}$ is the n-gravity field of interactions. There is no space in the geometrical sense between them. The force of the field between any two $preons^{(-)}$, anywhere in the Universe, is equal to one $\boldsymbol{g}^{-}$.

Figure 1 is a two-dimensional representation of quantum-geometrical space. The green circle represents a $preon^{(-)}$ arbitrarily chosen as origin and the blue circles represent $preons^{(-)}$ which

$$d = 1$$

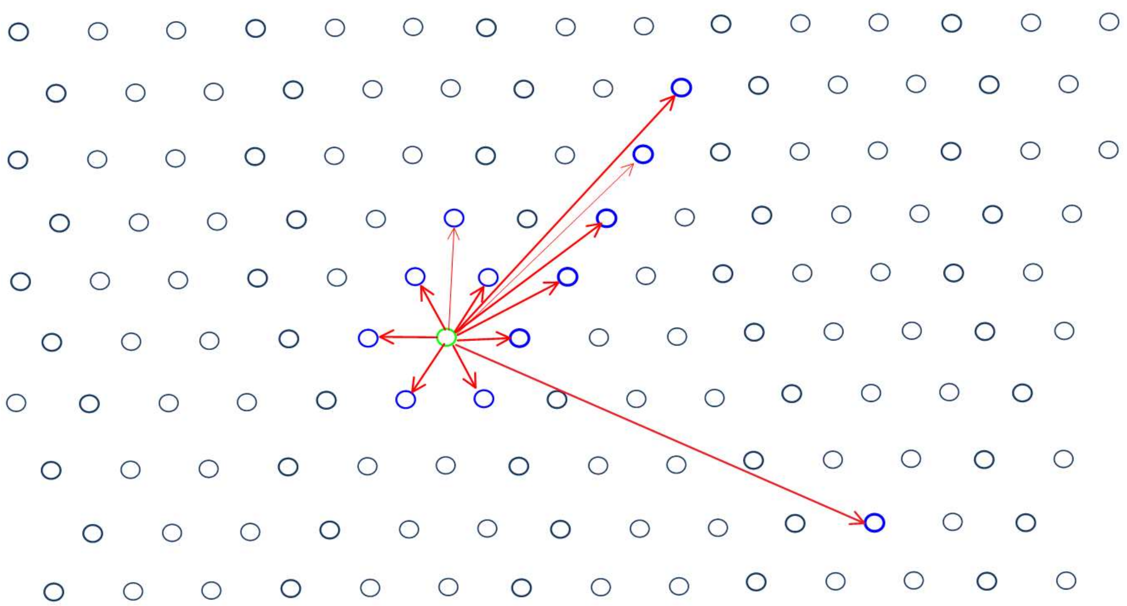

Figure 1

are all at one unit of distance from it. As we can see, distance in quantum-geometrical space at the fundamental scale is very different than Euclidian distance (though we will show below that Euclidian geometry emerges from quantum-geometrical space at larger scales).

Quantum-geometric space is not merely mathematical or geometrical but physical. Because of that, to distinguish it from quantum-geometrical space, we will refer to space in the classical sense of the term as *Euclidian space*.

Quantum-geometric space is very different from metric space. A consequence of this is that the distance between any two $preons^{(-)}$ in quantum-geometric space is be very different from the measure of the distance using Euclidian space; the distance between two points or $preons^{(-)}$ being equal to the number of leaps a $preon^{(+)}$ would need to make to move from one to the other.

In order to understand quantum-geometric space, one must put aside the notion of continuous infinite and infinitesimal space. Quantum-geometrical space emerges from the n-gravity interactions between $preons^{(-)}$. What that means is that $preons^{(+)}$ do not exist in space, they are space. Since $preons^{(-)}$ are fundamental and since QGD is founded on the principle of strict causality (this will be discussed in detail later), then the n-gravity field between $preons^{(-)}$ has always existed and as such may be understood as instantaneous. N-gravity does not propagate. It simply exists.

Figure 2 shows another example of how the distance between two $preons^{(-)}$ is calculated. So, although the Euclidian distance between the green $preon^{(-)}$ and any one of the blue $preons^{(-)}$ are nearly equal, the quantum-geometrical distances between the same varies greatly.

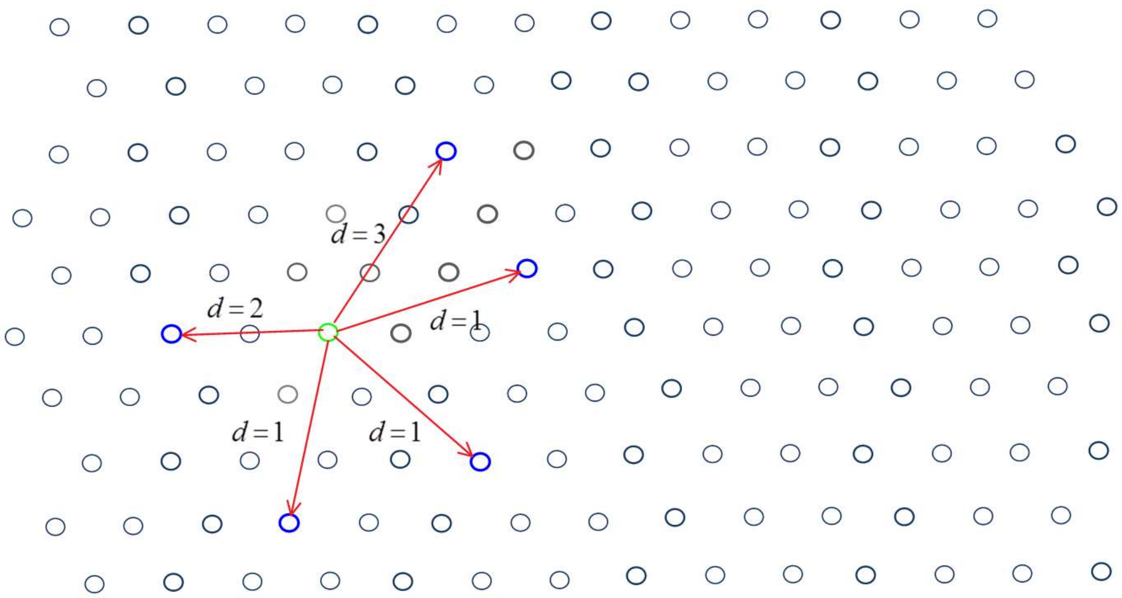

Figure 2

Since the quantum-geometrical distances do not correspond to the Euclidian distances, the theorems of Euclidean geometry do not hold at the fundamental scale. Trying to apply

Pythagoras's theorem to the triangle which in the figure 3 below defined by the blue, the red and the orange lines, we see that $a^2 + b^2 \neq c^2$.

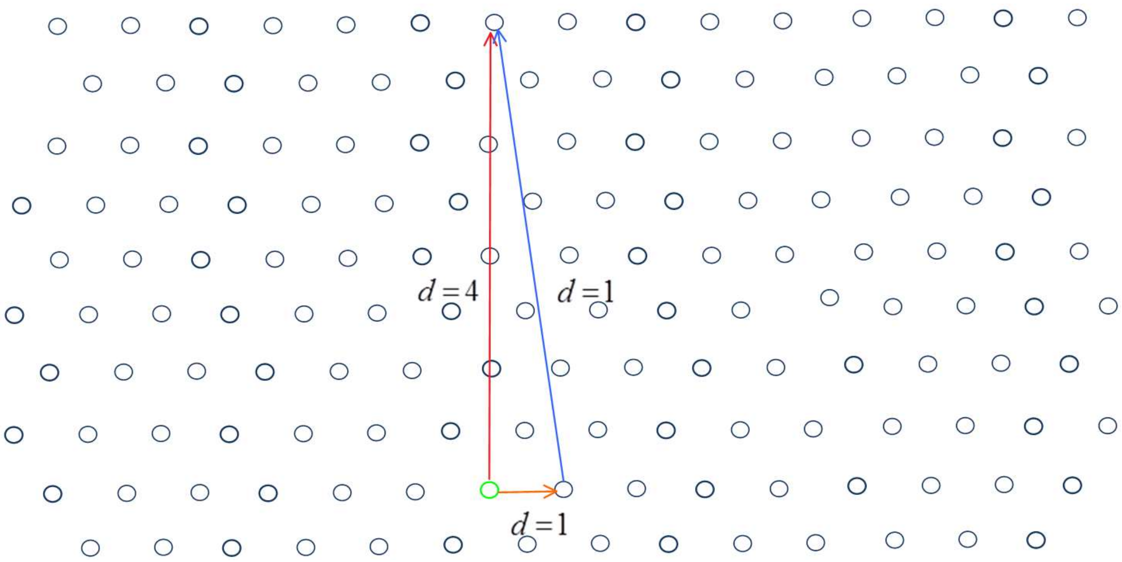

Figure 3

Also interesting in the figure 3 is that if a is the orange side, b the red side and c the blue side (what would in Euclidian geometry be the hypotenuse, then $a+c<b$. That is, the shortest distance between two $preons^{(-)}$ is not necessarily the straight line.

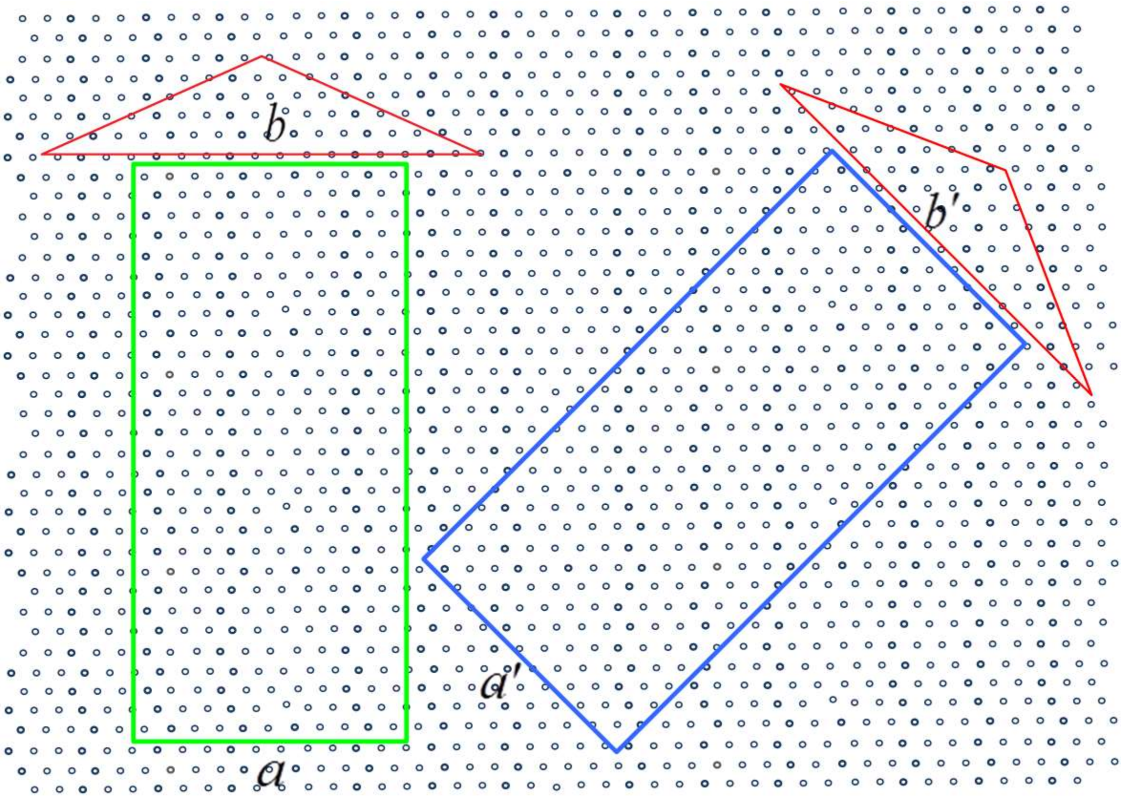

Figure 4

But we evidently live on a scale where Pythagoras's theorem holds, so how does Euclidian geometry emerge from quantum-geometrical space? Figure 4 shows the quantum-geometrical space two identical objects scan when moving in different directions.

Here, if we consider that the area in the blue rectangles is made of all the $preons^{(-)}$ through which the object moves, we see that as we move to larger scales, the number $preons^{(-)}$ contained in the green rectangle approaches the number of $preons^{(-)}$ in the blue rectangle, so that if the distance from a to b or from a' to b' is defined by the number of $preons^{(-)}$ contained in the respective rectangles divided by the width of the path, we find that $a \to b \simeq a' \to b'$.

Theorem on the Emergence of Euclidian Space from Quantum-Geometrical Space

If d and d_{Eu} are respectively the quantum-geometrical distance and the Euclidean distance two $preons^{(-)}$, then $\lim_{d\to\infty} d - d_{Eu} = 0$.

The theorem implies that beyond a certain scale the Euclidian distance between two points becomes a good approximation of the quantum-geometrical distance, but that below that scale, the closer we move towards the fundamental scale, the greater the discrepancies between the Euclidian and quantum-geometrical measurements of distance. A direct consequence of the structure of space and the derived theorem is that Euclidean geometric figures are ideal objects that though they can be conceptualized in continuous space can only be approximated in quantum-geometrical space to the resolution corresponding to the fundamental unit of distance.

It is important to note that since there are no infinities in QGD, the infinite sign ∞ is an impossibly large distance, hence the difference between quantum-geometrical and Euclidean distances, though it can very large or insignificantly small, can never be infinite or even equal to zero.

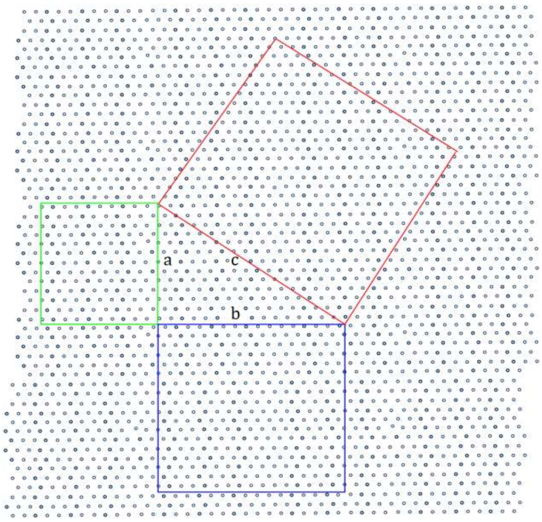

Figure 5

In figure 5, if n_1, n_2 and n_3 are respectively the number of parallel trajectories that sweep the squares a , b and c , for $n_{1-3} > M$, then $\overline{a} \approx \frac{\sum_{i=1}^{n_1} d_i}{n_1}$, $\overline{b} \approx \frac{\sum_{i=1}^{n_2} d_i}{n_2}$ and $\overline{c} \approx \frac{\sum_{i=1}^{n_3} d_i}{n_3}$ so that $\overline{a}^2 + \overline{b}^2 \approx \overline{c}^2$. Hence, given the quantum-geometrical length of the sides of any two of the three squares above, Pythagoras's theorem can be used to calculate an approximation of a the length of the side of the third. Also, the greater the values of n_1 , n_2 and n_3 the closer the approximation will be to the actual unknown length. That is $\lim\limits_{\substack{n_1\to\infty \\ n_2\to\infty \\ n_2\to\infty}} \left(\overline{a}^2 + \overline{b}^2\right) = \overline{c}^2$.

Application of the Theorem of Emergent Space

Even though reality at the fundamental scale is discrete, the theorem of emergence of Euclidean space allows us to use of continuous mathematics to describe dynamic systems at larger scales. We must however keep in mind that however accurate they may be, calculations using continuous mathematics remain approximations of the behaviour of the discrete components that form dynamics systems taken as a group and that quantum-geometrical reality only admits integer values of physical properties.

Interactions between Preons$^{(-)}$

We mentioned earlier that the interactions between two adjacent $preons^{(-)}$ is repulsive and the fundamental unit of n-gravity. Two $preons^{(-)}$ are adjacent if there is no other $preons^{(-)}$ between them. So for two $preons^{(-)}$, a and b , $G(a;b)=1\ g^{-}$ where $G(a;b)$ is the magnitude of the n-gravity interaction between them.

To obtain the magnitude of the n-gravitational interaction between any two $preons^{(-)}$ a and b , we need to take into account the interactions with and between the $preons^{(-)}$ that lie on the line of force connecting them. Thus, we need to count the number of interactions. Using the simple combinatory formula, we find that the magnitude of the n-gravitational interaction between any two $preons^{(-)}$ is

$$G^{-}(a;b)=\frac{d^2+d}{2}g^{-} \quad (1)$$

where d is the distance measure in number of $preons^{(-)}$ between a and b .

We will show in a later section that the repulsive force between space and matter is consistent with the effect we attribute to dark energy.

Properties of Preons$^{(-)}$

$Preons^{(-)}$ do not exist in space, they are space. This implies since any motion would imply that they would themselves be in space, which would contradict the 1st axiom, then they must be static.

And since they are fundamental, $preons^{(-)}$ do not decay into other particles the number of $preons^{(-)}$ is finite and constant which implies that quantum-geometrical space is finite, and that the Universe is finite.

Emerging Space and the Notion of Dimensions

We think of spatial dimensions as if they were physical in the way matter and space are physical, but the concept of dimensions is a relational concept which allows us to describe of the motion (even if that motion is nil) of an object or set of objects a relative to an object or set of objects b taken as a reference. Different systems of reference having directions and speeds relative to a given object or set of objects give different measurements of their positions, speed, mass and momentum and, according to dominant physics theories, there is no way to describe the motion of a reference system relative to space (or absolute motion), thus no way to know anything but relative measurements of properties are such as mass, energy, speed, momentum or position.

However, if QGD is correct in its description of space, then each fundamental unit of space is a distinct permanent position relative to all other discrete components of space ($preons^{(-)}$ being static) so that quantum-geometrical space can be taken as an absolute reference system which

The dimensionality of quantum-geometrical space (physical space) is the maximum number of elements in a set of non-concurrent and mutually orthogonal lines that have a common a $preon^{(-)}$. Space being an emergent property of $preons^{(-)}$ and all $preons^{(-)}$ having identical fundamental intrinsic properties, and all interacting to create space, then space must be isotropic.

Conservation of Space

That quantum-geometrical space is not infinitesimal also implies that geometric figures are not continuous either. For example, a circle in quantum-geometric space is a regular convex polygon whose form approaches that of the Euclidian circle as the number of $preons^{(-)}$ defining its vertex increases. That is, the greater the diameter of the polygon, the more its shape approaches that of the Euclidean circle (a similar reasoning applies for spheres).

The circumference of a circle in quantum-geometric space is equal to the number of triangles with base equal to 1 leap which form the perimeter of the polygon. It can also more simply be defined as the number of $preons^{(-)}$ corresponding to the polygon's vertex.

Since both the circumference of a polygon and its diameter have integer values, the ratio of the first over the second is a rational number. That is, if we define π as the ratio of the circumference of a circle over its diameter, then π is a rational function of the circumference and diameter of a regular polygon.

This implies that in quantum-geometric space the calculation of the circumference or area of a circle or the surface or volume of the sphere can only be approximated by the usual equations of Euclidian geometry.

The surface of a circle would be equal to the number of $preons^{(-)}$ within the region enclosed by a circular path.

From the above we understand that π , the ratio of the circumference of a circle over its diameter, is not a constant as in Euclidean geometry, but a function. If $\pi\left(a\right)$ is the proportionality function between the apothem a of the polygon and its perimeter then, since the base of the triangles that form the perimeter is equal to 1, it follows that the size of the polygon increases the value of the apothem of the polygon approaches the value of its circumradius and $\pi\left(a\right)$ approaches the geometrical value of π . Note that the smallest possible circumradius is equal to 1 leap, which defines the smallest possible circle which has six vertexes. Since in this case $2\pi r = 6$ and $r = 1$ it follows that $\pi(1) = 3\,\pi\left(1\right) = 3$.

$$\pi\left(a\right) = n / 2a$$

$$\lim_{a\to\infty} \pi\left(a\right) = \pi$$

where n is the number of sides of the polygon and ∞ is a very large number of the order of the quantum-geometrical diameter of a circle at our scale (QGD doesn't allow infinities).

So, within quantum-geometrical space, the geometrical π is a rational number that corresponds to the ratio of two extremely large integers. In fact, the size of the numerator and denominator are such that the decimal periodicity of their ratio is too large for any current computers to express.

Mathematical operations in quantum-geometry always are carried out from discrete units and can only result in discrete quantities.

In conclusion, the reader will understand that if space quantum-geometrical, then the mathematics used to describe it and the objects it contains must also be quantum-geometrical. Continuous mathematics, though it can provide approximations of discrete phenomena at larger than fundamental scales, becomes inadequate the closer we get to the fundamental scale.